

**THE MARKET SHARE RULE WITH PRICE EROSION: PATENT
INFRINGEMENT LOST PROFITS DAMAGES AFTER CRYSTAL**

*Roy J. Epstein**

I.	INTRODUCTION	3
II.	PERLS AND LOST PROFITS	8
	A. <i>Overview of Patent Infringement Damages Concepts</i>	8
	B. <i>The Market Share Rule and PERLS</i>	14
	C. <i>The Split Award</i>	22
III.	ESTABLISHING THE AMOUNT OF PRICE EROSION	23
	A. <i>Case Precedents</i>	24
	B. <i>An Economic Model for Price Erosion</i>	27
IV.	APPLICATION OF PERLS TO SELECTED DECISIONS	31
	A. <i>Micro Motion, Inc. v. Exac Corp.</i>	32
	B. <i>Minnesota Mining & Mfg. Co. v. Johnson & Johnson Orthopaedics</i>	34
	C. <i>Crystal v. TriTech</i>	36
V.	CONCLUSION	40

* © 2003 Roy J. Epstein, all rights reserved. Roy Epstein is an economic consultant in the Boston, Massachusetts area. Dr. Epstein is an ABA Associate Member and earned a B.A. from Wesleyan University in 1977 and a Ph.D. in 1984 from Yale University. He has previously published *A HISTORY OF ECONOMETRICS* (North-Holland 1987). Dr. Epstein thanks Steve Brenner, Michael Einhorn, Darin Lee, and especially Fred Knapp for helpful comments on an earlier draft. The author also thanks Judge A. David Mazzone, Judge Randall Rader, and Asim Varma for their interest and encouragement. Dr. Epstein may be contacted at rje@royepstein.com. All opinions and errors are the responsibility of the author.

APPENDIX

I.	PRICE ELASTICITY AND CHANGES IN REVENUE	42
II.	THE CONVENTIONAL MARKET SHARE RULE	43
III.	THE GENERAL PERLS EQUATION FOR LOST PROFITS DAMAGES ...	43
IV.	JUDGE EASTERBROOK'S METHOD IN <i>MAHURKAR</i> AS A SPECIAL CASE OF PERLS	45
V.	PRICE EROSION AND THE ALMOST IDEAL DEMAND SYSTEM	46

LIST OF FIGURES

1.	Figure 1. Hypothetical Actual Market with Infringement	18
2.	Figure 2. Hypothetical But-For Market	19
3.	Figure 3. Comparison of Conventional and PERLS Lost Profits	20

LIST OF TABLES

1.	Table 1. Alternative Damages Scenarios in <i>Crystal</i>	39
----	--	----

I. INTRODUCTION

What is the best way to determine the lost profits damages caused when one firm infringes another's patent? The answer to this question has gradually evolved in court decisions and legal scholarship, reflecting a continuing effort to craft a solution satisfactory to both the demands of the law and the laws of economics. This article focuses on two areas in the frontier of thinking in lost profits damages: price erosion and the "split award," which includes both lost profits and a "reasonable royalty" on infringing sales. In both these areas, economic reasoning can make significant further contributions towards improving the reliability of lost profits awards. The analysis is also timely since the number of patent infringement cases continues to increase with a rising incidence of lost profit and split awards. This article extends and simplifies previous approaches, building on legal precedents while incorporating an improved and understandable economic framework.

The prevailing method for calculating lost profits damages in patent infringement originated with *State Industries v. Mor-Flo*¹ in the late 1980s. That decision established both the "market share" rule for lost sales and use of the split award.² The market share rule considers that portion of the patent holder's relevant market that has been captured by the infringing

¹ *State Indus. v. Mor-Flo Indus.*, 883 F.2d 1573, 1576, 12 U.S.P.Q.2d (BNA) 1026, 1027 (Fed. Cir. 1989), *aff'd*, 948 F.2d 1573, 20 U.S.P.Q.2d (BNA) 1738 (Fed. Cir. 1991). The market share theory was born when the district court accepted the testimony of patent holder State Industries' vice-president of marketing, who used State Industries' market share during the infringement to determine the fraction of the defendant's competing, infringing sales that should be considered lost.

² *Id.* at 1577, 1580, 12 U.S.P.Q.2d (BNA) at 1028, 1031.

firm.³ In the simplest two-firm model the market share rule assumes that the patent holder would capture all the infringing revenue.⁴

When the market also includes non-infringing alternatives, the market share rule divides the infringing sales among the patent holder and the non-infringing firms in proportion to their respective market shares.⁵ The split award includes a reasonable royalty to the patent holder for the infringing sales allocated to the non-infringing firms (the split award is not used with a two-firm market).⁶ The logic of *State Industries* therefore treats each infringing sale in the relevant market as either a lost sale for the patent holder or a basis for a royalty payment.⁷

This framework is open to serious economic criticism when price erosion damages are included in the lost profits calculation. First, by definition, price erosion means that prices in the “but-for” market (i.e., the market without the infringing sales) would be higher. It is not always recognized, however, that the higher prices generally imply a reduction in demand, with a corresponding reduction in the amount of lost sales.⁸ The market share rule as enunciated in *State Industries* does not address this “price elasticity” effect because that case did not include a claim of price

³ *Id.* at 1578, 12 U.S.P.Q.2d (BNA) at 1029.

⁴ *Id.*

⁵ *Id.*

⁶ *Id.* at 1577, 12 U.S.P.Q.2d (BNA) at 1028.

⁷ *Id.*

⁸ *See* *Crystal Semiconductor Corp. v. Tritech Microelectronics Int’l, Inc.*, 246 F.3d 1336, 1359, 57 U.S.P.Q.2d (BNA) 1953, 1966 (Fed. Cir. 2001).

erosion. Failure to account for the price elasticity effect will overstate lost profits.⁹

Second, a complete analysis of lost profits, which would include lost sales and price erosion, should be sufficient to compensate the patent holder for the economic loss caused by the competing, infringing sales.¹⁰ The additional reasonable royalty on infringing sales in the same market through the split award is likely to overcompensate the patent holder.

The Court of Appeals for the Federal Circuit (“Federal Circuit”) has recently indicated increased awareness of the need for more sophisticated economic analysis of lost sales with price erosion, but it has not yet revisited the justification for the split award.¹¹ The court strongly signaled in *Crystal v. TriTech* that the common practice of asserting price erosion independently of the lost sales claim is not credible due to elasticity and is unlikely to be acceptable in the future:

According to basic tenets of economics, because Crystal is in a competitive market, if Crystal raised prices, Crystal’s sales would have fallen. . . . A patentee must produce credible economic evidence to show the decrease in sales, if any, that would have occurred at the higher hypothetical price. . . . To prevent inconsistent results, this court will not venture to evaluate price erosion and lost profits damages separately. .

⁹ See *id.*

¹⁰ See *Panduit Corp. v. Stahl Bros. Fibre Works, Inc.*, 575 F.2d 1152, 197 U.S.P.Q. (BNA) 726 (6th Cir. 1978).

¹¹ See *Crystal Semiconductor*, 246 F.3d 1336, 57 U.S.P.Q.2d (BNA) 1953.

. . . Crystal cannot have both lost profits and price erosion damages on each of those lost sales.¹²

To illustrate the Federal Circuit's concern, suppose that in a market for widgets a patent holder sold 800 units and an infringer, the sole competitor, sold 200 units and that the actual price was \$10. The patent holder therefore had revenue of \$8,000, the infringer had revenue of \$2,000, and the total market size was \$10,000. Assume the infringement caused 10% price erosion (i.e., in the but-for market the patent holder would have charged \$11). Finally, assume there was a 40% profit margin on each actual sale. The Federal Circuit rejected the plaintiff's theory in *Crystal* that implied damages of \$1,800, calculated as follows: 10% price erosion times \$10,000 total market revenue (\$1,000) plus the product of 40% profit margin times \$2,000 "lost" sales (\$800) equals \$1,800.

The problem is, due to elasticity, at the higher but-for price the patent holder would not have been able to sell the 1,000 widgets that underlie this calculation. This would imply smaller damages. The Federal Circuit's reasoning concerning lost profits in *Crystal* is fine as far as it goes, but it leaves open the question of how to reach an economically consistent result. Furthermore, the court does not make the link to the split award.

This article offers a simple method to integrate lost sales with the effects of price erosion in order to calculate economically consistent lost profits damages. This integrated approach will be denoted PERLS (Price ERosion and Lost Sales). PERLS, in accordance with principles repeatedly laid out in the leading cases, calculates damages as the difference between the patent holder's profits in the but-for market and the actual market.¹³

¹² *Id.* at 1359-61, 57 U.S.P.Q.2d (BNA) at 1966-68.

¹³ *Aro Mfg. Co. v. Convertible Top Replacement Co.*, 377 U.S. 476, 507, 141 U.S.P.Q. (BNA) 681, 694 (1964) ("[In patent infringement cases t]he question to be asked in determining damages is 'how much the patent holder and licensees suffered by the infringement', which is primarily

PERLS requires a single additional parameter, the market price elasticity of demand, to insulate the analysis from the pitfalls identified by the Federal Circuit. Mathematically, PERLS is equal to the conventional implementation of the market share rule plus a single adjustment for price elasticity.

PERLS stands in a direct line of development of economic tools to analyze lost profit damages.¹⁴ The existing market share rule is equivalent to the special case of PERLS with no price erosion. Moreover, the insightful analysis offered by Judge Frank Easterbrook in *Mahurkar* is also a special case of PERLS with a price elasticity of demand equal to -1 and no non-infringing competitors.¹⁵ The PERLS damages calculation builds on these precedents in a tractable manner using data typically available in litigation. By drawing on standard economic principles that have been used in numerous court proceedings, PERLS should yield damages that meet the standard of disciplined, “credible economic evidence” sought by the Federal Circuit.

Part II explains the rationale for the PERLS lost profits calculation, compares PERLS to the market share rule and *Mahurkar*, and criticizes the

the amount the patent holder-licensee would have made if there had been no infringement.”) (quoting *Livesay Window Co. v. Livesay Indus., Inc.*, 251 F.2d 469, 471 (5th Cir. 1958)).

¹⁴ An alternative methodology for lost profits with price erosion that bears some similarities to PERLS was recently proposed by Gregory J. Werden et al., *Economic Analysis of Lost Profits from Patent Infringement With and Without Noninfringing Substitutes*, 27 AIPLAQ.J. 305 (1999). An integrated price erosion and lost sales calculation that does not rely on the market share rule was introduced in Roy J. Epstein, *State Industries and Economics: Rethinking Patent Infringement Damages*, 9 FED. CIR. B.J. 367 (2000).

¹⁵ *In re Mahurkar Double Lumen Hemodialysis Catheter Patent Litig.*, 831 F. Supp. 1354, 28 U.S.P.Q.2d (BNA) 1801 (N.D. Ill. 1993), *aff'd*, 71 F.3d 1573, 37 U.S.P.Q.2d (BNA) 1138 (Fed. Cir. 1995).

use of the split award with price erosion. Part III discusses methods of determining the amount of price erosion. Part IV provides examples of the new methodology using data from several important decisions. Part V contains brief concluding remarks. The appendix includes supporting mathematical details.

II. PERLS AND LOST PROFITS

A. *Overview of Patent Infringement Damages Concepts*

An appropriate damages award at least makes the patent holder whole; that is, an award should at least equal the difference between the patent holder's expected profits but-for the infringement and the actual, presumably lower, profits. The patent statute does not specify a damages methodology for this purpose, stating only that "[u]pon finding for the claimant the court shall award the claimant damages adequate to compensate for the infringement, but in no event less than a reasonable royalty for the use made of the invention by the infringer, together with interest and costs as fixed by the court."¹⁶ A reasonable royalty is therefore the floor on patent infringement damages, but plaintiffs may bring additional claims for lost profits. The landmark opinion in *Panduit v. Stahlin Bros. Fibre Works* initiated the modern analysis of lost profits in this context by introducing its famous four-pronged test whereby "a patent owner must prove: (1) demand for the patented product, (2) absence of acceptable non-infringing substitutes, (3) his manufacturing and marketing capability to exploit the demand, and (4) the amount of the profit he would have been

¹⁶ 35 U.S.C. § 284 (2000).

made.”¹⁷ Many commentaries in the past few years have detailed the relevant case history and the emergence of market share damages.¹⁸

Lost profits are measured as the difference between the profits the patent holder would have earned but for the infringing sales and the profits the patent holder actually earned.¹⁹ As will be explained below, a lost profits analysis using PERLS consists of several distinct elements. First is market definition, which essentially identifies the competition facing the patent holder, including the infringer and non-infringing alternatives. The market definition is used to measure revenue and shares for the firms in the market, as well as other indicators of competition.²⁰ Second is the amount of price erosion caused by the illegal competition. Third is the market price elasticity of demand. Price elasticity is the percent decrease in quantity demanded due to a 1% increase in price, holding all else equal.²¹ In the but-for market, the price is proportionally higher by the amount of the price

¹⁷ *Panduit Corp. v. Stahl Bros. Fibre Works, Inc.*, 575 F.2d 1152, 1156, 197 U.S.P.Q. (BNA) 726, 730 (6th Cir. 1978).

¹⁸ *See, e.g.*, Paul M. Janicke, *Contemporary Issues in Patent Damages*, 42 AM. UNIV. L. REV. 691 (1993); Allan N. Littman, *Monopoly, Competition and Other Factors in Determining Patent Infringement Damages*, 38 IDEA 1 (1997); Joel Meyer, *State Industries v. Mor-Flo and the Market Share Approach to Patent Damages: What is Happening to the Panduit Test?*, 6 WIS. L. REV. 1369 (1991).

¹⁹ *Aro Mfg. Co. v. Convertible Top Replacement Co.*, 377 U.S. 476, 507, 141 U.S.P.Q. (BNA) 681, 694 (1964).

²⁰ The standard reference on market definition is U.S. DEP'T OF JUSTICE & FED. TRADE COMM'N, *HORIZONTAL MERGER GUIDELINES*, 57 Fed. Reg. 41,552 (Apr. 2, 1992), available at <http://www.usdoj.gov/atr/public/guidelines/hmg.htm> (last visited Nov. 15, 2002).

²¹ MICHAEL L. KATZ & HARVEY S. ROSEN, *MICROECONOMICS* 74 (3d ed. 1998).

erosion, and the quantity demanded falls by the amount of the price erosion multiplied by the price elasticity.²² Fourth is the patent holder's incremental cost. Incremental cost and the amount of price erosion imply the patent holder's incremental profit margins in the actual and but-for markets. The following paragraphs review these concepts in more detail.

Market definition is essential for PERLS (and any other approach based on market share) because it identifies the competitors that must be considered to measure economically meaningful shares in the patent holder's market; overly narrow markets tend to overstate damages and conversely for overly broad markets. Market definition establishes the range of products against which the patent holder competes as well as the geographic area in which the patent holder makes, or would have made, sales.²³ Economic techniques to define markets have long been used in antitrust litigation and the same principles can be applied to patent damages.²⁴ As a threshold matter, if the patent holder and the infringer do not compete in the same market, economic principles make a claim for lost profits doubtful and shift the focus to determination of a reasonable royalty.²⁵ Moreover, as Judge Easterbrook pointed out in *Grain Processing v. American Maize-Products*, the evidence on market definition may indicate that the infringer would have been a but-for competitor anyway, perhaps by using other available technology or by designing around the patent, with potentially critical implications for the amount of lost profits.²⁶

²² See *id.*

²³ See *id.*

²⁴ U.S. DEP'T OF JUSTICE & FED. TRADE COMM'N, *supra* note 20, at 4.

²⁵ See *BIC Leisure Prods., Inc. v. Windsurfing Int'l, Inc.*, 1 F.3d 1214, 1218-19, 27 U.S.P.Q.2d (BNA) 1671, 1674-75 (Fed. Cir. 1993).

²⁶ See *Grain Processing Corp. v. Am. Maize-Prods. Co.*, 893 F. Supp. 1386, 1392, 37 U.S.P.Q.2d (BNA) 1299, 1303 (N.D. Ind. 1995).

The relevance of market definition can extend beyond the patented product. One example is to identify “convoyed sales” of unpatented products that can be sold with the patented product. For example, suppose a manufacturer of patented sunglasses tended to sell them with an unpatented eye strap (in economic terms, the sunglasses and the eye strap are “complements”). An infringer may deprive the patent holder of both sales. In *Rite-Hite*, the patent holder sold an unpatented alternative that was a substitute for the patented product.²⁷ The infringer was found to cause lost sales in both categories.²⁸ The reasoning in such cases appears to take a more inclusive view of lost sales that should bring awards closer to a “make whole” standard in terms of economics.²⁹

The Federal Circuit highlighted the centrality of market definition in *BIC Leisure v. Windsurfing International*.³⁰ The patent holder manufactured high-performance sailboards that sold at retail for about \$600.³¹ The infringing sailboards were recreational models that sold for about \$350.³² Claiming lost profits, the patent holder essentially argued that all sailboards belonged in a single market and that the patent holder was entitled to its

²⁷ *Rite-Hite Corp. v. Kelly Co.*, 56 F.3d 1538, 1543, 35 U.S.P.Q.2d (BNA) 1065, 1067 (Fed. Cir. 1995).

²⁸ *Id.*

²⁹ PERLS can be applied to both convoyed sales and *Rite-Hite* issues, but a detailed analysis is beyond the scope of the present article.

³⁰ *BIC Leisure Prods., Inc. v. Windsurfing Int’l, Inc.*, 1 F.3d 1214, 27 U.S.P.Q.2d (BNA) 1671 (Fed. Cir. 1993).

³¹ *Id.* at 1216-18, 27 U.S.P.Q.2d (BNA) at 1673-74.

³² *Id.*

“market share” of the infringing sales.³³ The court was not convinced that buyers of recreational boards would have chosen the patent holder’s high performance product, particularly because numerous sellers of inexpensive, non-infringing sailboards would remain in the but-for market.³⁴ That is, the infringer sold in a different market.³⁵ Thus, lost profits were not awarded.³⁶

Price erosion enters PERLS in several places. The higher but-for price generally implies that the patent holder suffered diminished profitability on its actual sales.³⁷ In addition, the profit margins on the incremental sales that would be gained in the absence of the infringement should be higher than the patent holder’s actual margins (assuming unchanged costs).³⁸ These effects of price erosion increase the amount of lost profits. Price erosion also implies a reduction in the quantity sold in the but-for market through price elasticity. This effect, which preoccupied the Federal Circuit in *Crystal*, lowers the amount of lost profits.³⁹

³³ *See id.*

³⁴ *Id.*

³⁵ *See id.*

³⁶ *See* John C. Jarosz & Erin M. Page, *The Panduit Lost Profits Test After BIC Leisure v. Windsurfing*, 3 FED. CIR. B.J. 311 (1993).

³⁷ *See* KATZ & ROSEN, *supra* note 21, at 73-74; *Crystal Semiconductor Corp. v. Tritech Microelectronics Int’l, Inc.*, 246 F.3d 1336, 1359, 57 U.S.P.Q.2d (BNA) 1953, 1966 (Fed. Cir. 2001).

³⁸ Potential complications raised by the ability of the patent holder to price discriminate are not discussed in this article.

³⁹ *Crystal Semiconductor Corp. v. Tritech Microelectronics Int’l, Inc.*, 246 F.3d 1336, 1359, 57 U.S.P.Q.2d (BNA) 1953, 1966 (Fed. Cir. 2001).

Price elasticity is a fundamental concept when there is price erosion because higher prices almost always lead to reduced quantity demanded.⁴⁰ Many econometric tools are available to determine price elasticity empirically.⁴¹ PERLS uses price elasticity to analyze changes in the size of the market due to price erosion. If the percentage change in quantity exceeds the price erosion, but-for market revenue will fall, and vice versa.⁴² In the important special case of elasticity equal to -1 (used, for example, by Judge Easterbrook in *Mahurkar*) the two effects offset each other and market revenue is unaffected by price erosion.⁴³

Profitability should be calculated using incremental cost, which excludes the costs that would be incurred in both the actual and but-for markets.⁴⁴ Stated differently, the lost profits equal the difference in revenue

⁴⁰ KATZ & ROSEN, *supra* note 21, at 73-74.

⁴¹ See, e.g., Jonathan B. Baker & Daniel L. Rubinfeld, *Empirical Methods in Antitrust Litigation: Review and Critique*, 1 AM. L. AND ECON. REV. 386 (1999).

⁴² See *infra* app. § III.

⁴³ See *infra* app. § IV. For alternative discussions of price elasticity in the context of patent infringement see SUMANTH ADDANKI, *ECONOMICS AND PATENT DAMAGES: A PRACTICAL GUIDE* (Nat'l Econ. Res. Assoc., Working Paper No. 21, Nov. 1993), available at <http://www.nera.com/wwt/publications/3953.pdf> (last visited Oct. 10, 2002); James Gould & James Langenfeld, *Antitrust and Intellectual Property: Landing on Patent Avenue in the Game of Monopoly*, 37 IDEA 449 (1997); Richard A. Rapp & Phillip A. Beutel, *Patent Damages: Updated Rules on the Road to Economic Rationality*, in *PATENT LITIGATION 2000*, at 849 (PLI Patents, Copyrights, Trademarks, and Literary Property, Course Handbook Series No. 6-619, 2000), available at WL 619 PLI/Pat 849.

⁴⁴ *Paper Converting Machine Co. v. Magna-Graphics Corp.*, 745 F.2d 11, 22, 223 U.S.P.Q. (BNA) 591, 599 (Fed. Cir. 1984).

minus the difference in costs.⁴⁵ The difference in costs (whether expressed on a total or per unit basis) is the incremental cost concept. The major pitfall when measuring incremental cost is that many firms allocate both fixed costs (e.g., depreciation) and common costs (e.g., marketing and administration) to their internal accounts that might be used to calculate incremental costs. Accounting and statistical analyses are often required to remove these allocations to calculate profits properly.

B. The Market Share Rule and PERLS

Suppose that in a market with an infringer, the patent holder has a 60% market share, the infringer has a 25% market share, and non-infringing alternatives account for the remaining 15% market share. In dollar terms, suppose the total market was \$120 million, so that the patent holder has sales of \$72 million and the infringer has sales of \$30 million. Assume further that the patent holder has a 40% incremental profit margin (price minus incremental cost, divided by price) in the market with infringement.

This information is sufficient to calculate damages using the conventional market share rule. The patent holder would have a but-for market share of 80% (calculated as actual share of 60% divided by the total non-infringing share of 75%).⁴⁶ The lost sales would be \$24 million (80% times the infringing revenue of \$30 million), and the associated lost profits damages would equal \$9.6 million (40% times \$24 million).⁴⁷

⁴⁵ See *infra* app. §§ I, II.

⁴⁶ See *infra* app. § II.C.

⁴⁷ A split award would include a reasonable royalty on the remaining \$6 million (\$30 million minus \$24 million) of the infringing sales not included in the lost profits calculation.

Now suppose it is found that the infringement caused 10% price erosion and that the market price elasticity of demand equaled -1.5 . A naive application of the conventional market share rule would set lost profits equal to \$19.2 million, calculated as the sum of \$9.6 million in lost profits calculated previously plus \$7.2 million in price erosion on the patent holder's actual sales (10% price erosion times \$72 million) plus \$2.4 million in additional price erosion on the alleged lost sales (10% price erosion times \$24 million). Total lost profits would be doubled because such a calculation implicitly assumes a price elasticity of zero.⁴⁸ The *Crystal* decision rejected this approach because it contradicts basic economics.

PERLS, however, which explicitly accounts for the assumed price erosion and elasticity, yields lost profits of \$12.7 million. Price erosion increases lost profits in this example but by less than predicted by the naive method. The differences in the two approaches increase with greater amounts of price erosion and with greater magnitudes of price elasticity.

PERLS is a direct extension of the market share rule. Mathematically, PERLS is equal to the naive method just described plus a single adjustment for elasticity. The market share rule is embodied in PERLS in the following sense: both PERLS and the conventional approach incorporate the assumption that the patent holder's but-for market share is the quotient of its actual market share divided by the total share of non-infringing sales (e.g., 80% in the above example). PERLS then draws out the implication of this assumption in the presence of price erosion for any given value of price elasticity. Putting the elements together, the PERLS lost profits calculation (with no cost effects) can be expressed as:

$$\text{PERLS lost profits} = \text{Market share lost profits} + \text{the product of the price erosion \% times patent holder's revenue} + \text{the product of price erosion \% times market share lost sales} + \text{price elasticity adjustment}$$

Algebraically, the corresponding result is

⁴⁸ See *infra* app. § II.E.

PERLS lost profits =

$$\mu REV \frac{s_1}{1-s_1} + \delta REV + \delta REV \frac{s_1}{1-s_1} + \varepsilon \frac{REV}{1-s_1} \frac{\delta}{1+\delta} (\delta + \mu) \quad (\text{Equation 1})$$

where

μ = patent holder's actual incremental profit margin

REV = patent holder's actual revenue

s_1 = infringer's market share

δ = price erosion %

ε = market price elasticity of demand.

The only new parameter in PERLS relative to the naive market share approach is the price elasticity.⁴⁹ Economic principles indicate the mathematical form of the elasticity adjustment in the lost profits equation to ensure that lost profits equal the difference between but-for and actual profits. Observe that with zero price erosion ($\delta=0$), PERLS is equivalent to the conventional market share rule, which is a special case of PERLS.⁵⁰

In the example, the patent holder has a 40% incremental margin on revenue of \$72 million, the infringer has a 25% market share, price erosion is 10%, and the market price elasticity is -1.5 . Substituting these values into the expression yields the lost profits of \$12.7 million. It can be seen that the price elasticity adjustment lowers damages by \$6.5 million relative to the naive calculation of \$19.2 million.

⁴⁹ See *infra* app. § III.A (providing details and extension to the case of different but-for costs).

⁵⁰ See *infra* app. § IV (explaining why Judge Easterbrook's method in *Mahurkar* is also a special case of PERLS).

PERLS is illustrated graphically in Figures 1–3. Figure 1 shows the actual market with infringement. Specifically, the graph shows the size of the market (\$120 million) corresponding to the average price in the actual market. The market revenue curve depicts the effect of the assumed elasticity: at higher prices the market would be smaller. Figure 2 shows the but-for market. The average market price is 10% higher, and the market size contracts by 5% to \$114 million due to the price elasticity effect.⁵¹ The patent holder and the non-infringing alternatives have higher revenues due to higher market shares. The infringer is out of the market. Figure 3 presents the lost profits damages calculation. The lost sales and price erosion from the naive calculation are shown together with the PERLS elasticity adjustment. The sum of these elements is the total lost profits damages of \$12.7 million, which equals the difference between the but-for and actual profit.

To summarize, PERLS yields an economically consistent calculation of lost profits with price erosion. The required information includes the patent holder's revenue, the infringer's market share, the amount of price erosion, the price elasticity of demand for the market, the effect of the infringement on the patent holder's incremental unit costs, and the patent holder's profit margin based on variable costs. Economic, accounting, and statistical analyses all play a role in identifying the proper inputs for the lost profits calculation. For example, definition of the relevant market is necessary both to ensure that lost profits are calculated only for infringing sales that compete with the patent holder's sales and to determine total market revenue for computing the infringer's share. The amount of price erosion and the price elasticity of market demand are also economic questions. Adjustments to reported accounting data are typically required to obtain relevant measures of profit margins. PERLS provides a coherent structure for the overall analysis.

⁵¹ The 5% market revenue reduction equals 10% price erosion multiplied by 1 plus the elasticity of -1.5 . See *infra* app. § I.

Figure 1

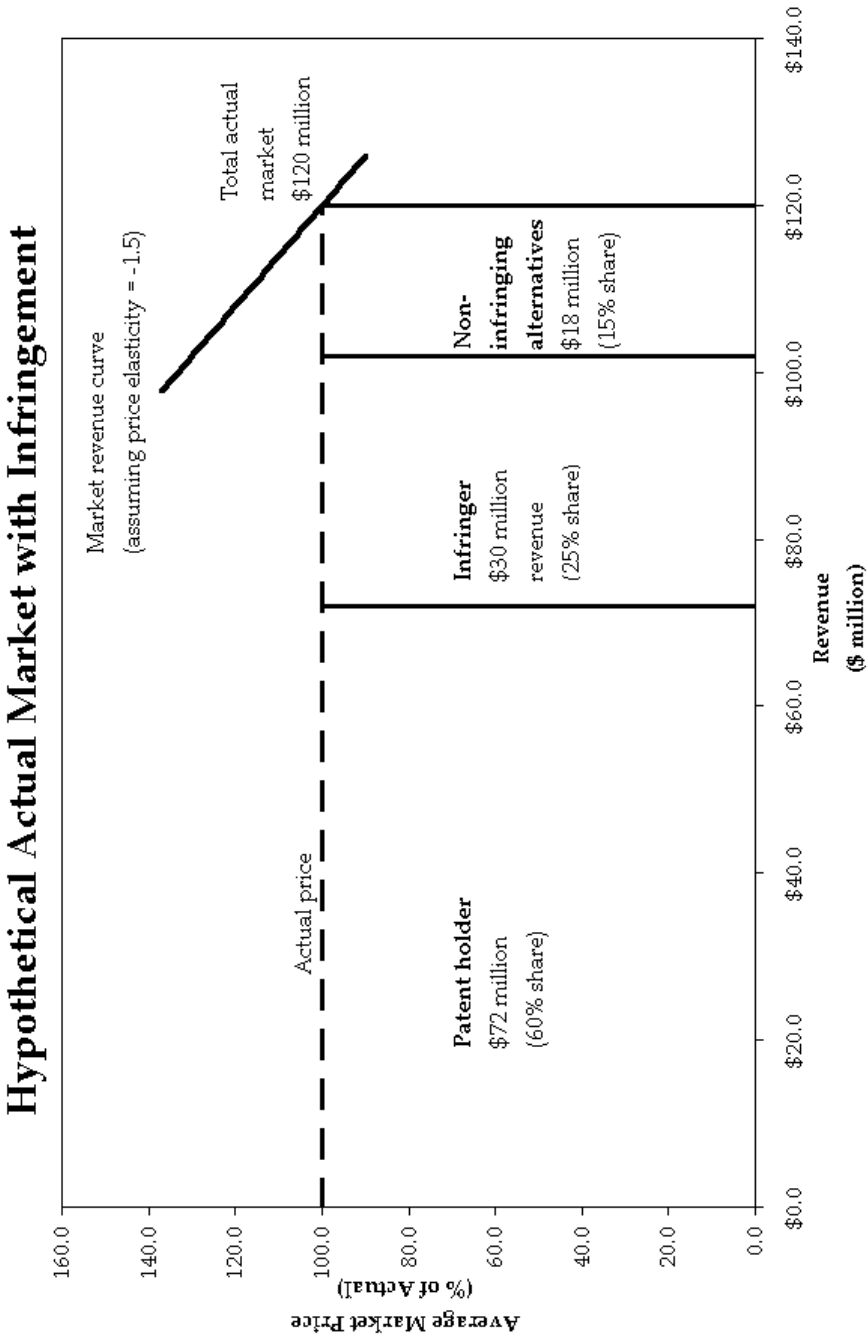


Figure 2

Hypothetical But-For Market

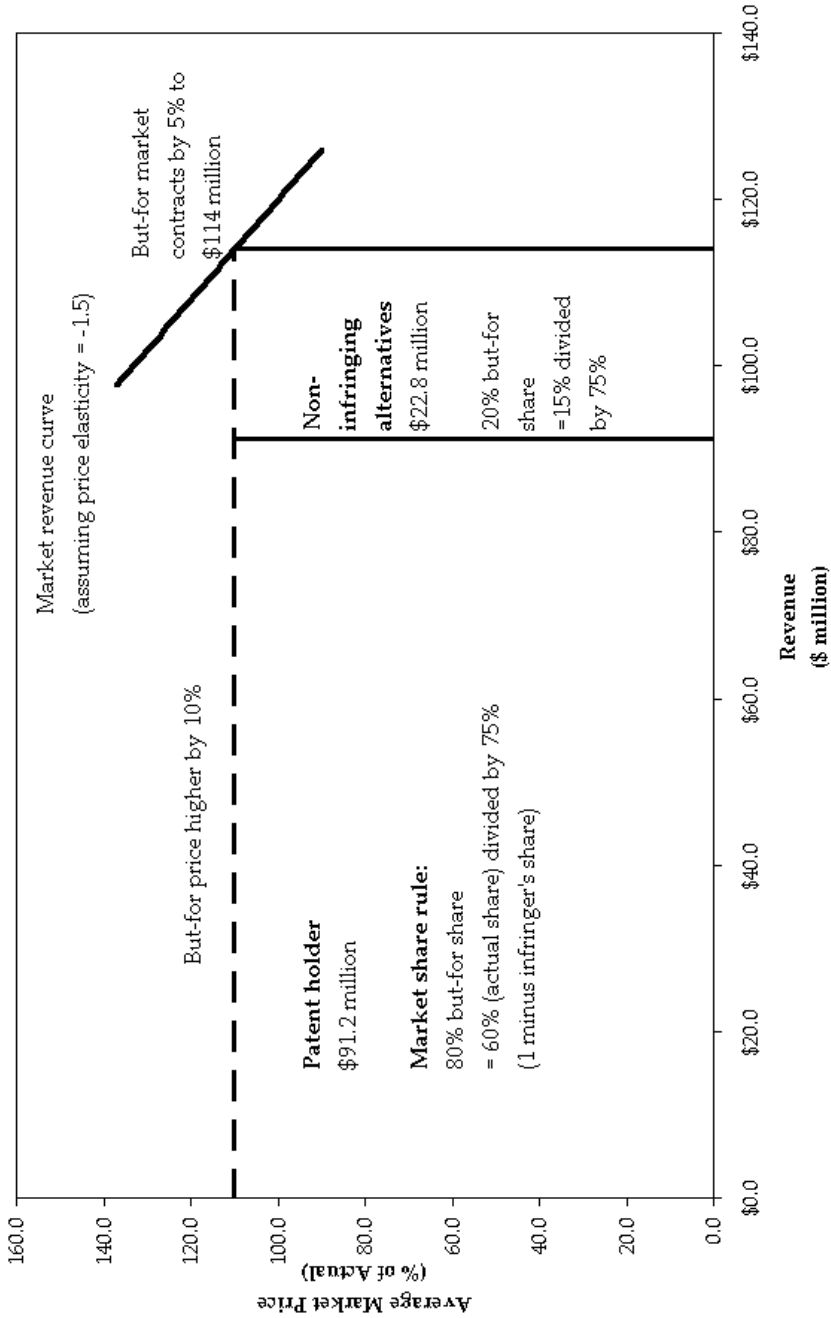
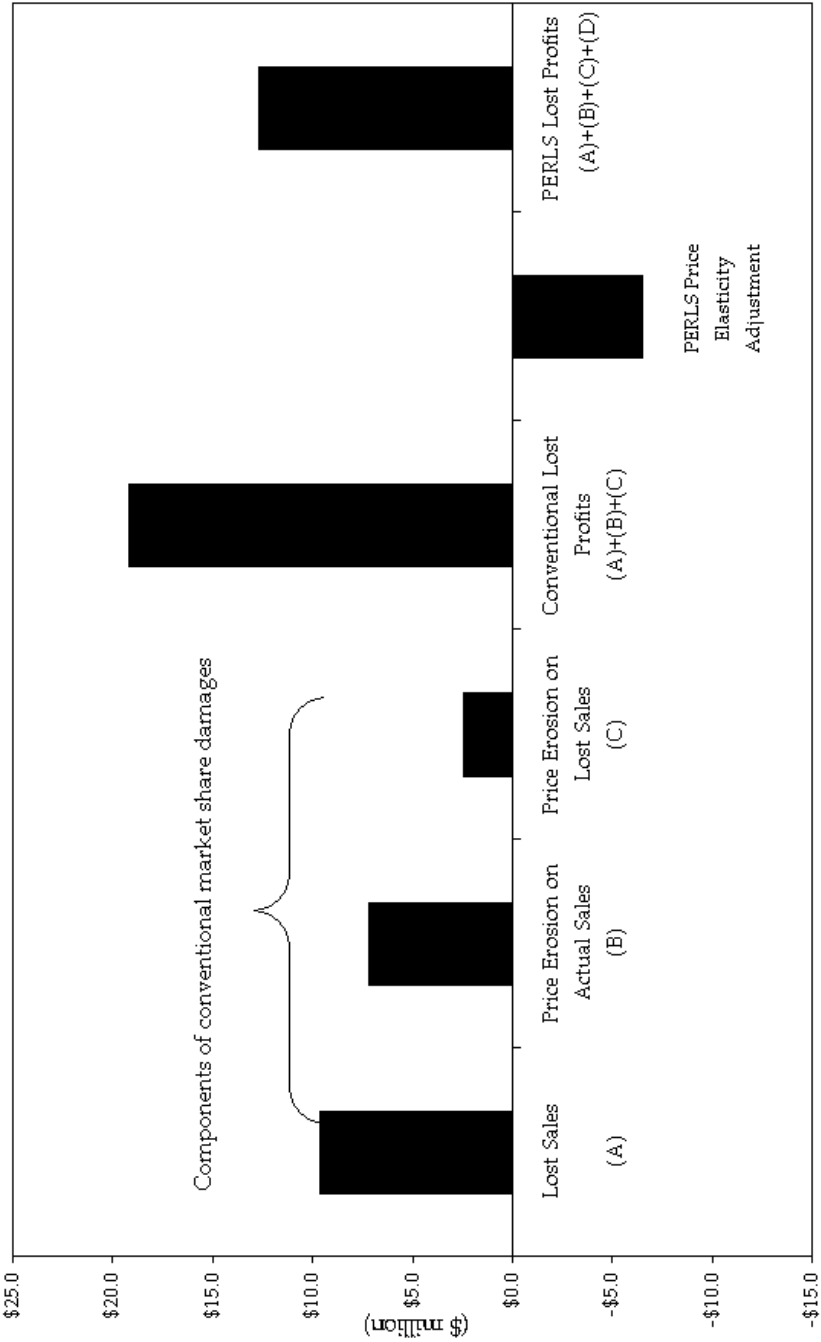


Figure 3

Comparison of Conventional and PERLS Lost Profits



The PERLS framework significantly strengthens the case for broader applicability of the market share rule, but it does not eliminate the need for other economic analysis. For example, if the economics of a particular case support Judge Easterbrook's view in *Grain Processing*, the infringer would remain in the market and the market share approach would not apply (and damages would likely be greatly reduced).⁵² There is also a potential issue when competing products are highly differentiated or firms have very different costs. The assumption that the infringing sales would be distributed to the other firms in proportion to their market share may not always be accurate in this situation.⁵³

PERLS is amenable to further refinement, albeit at some increase in complexity. In particular, it is possible to generalize the market share rule to allow a more flexible representation of the diversion of the infringer's sales to the other firms in the market.⁵⁴ It is also possible to take account of issues related to convoyed sales. The appeal of the market share rule, apart from its intuitive reasonableness, is that PERLS implements it in a way that comports with basic economic principles and with a minimal amount of data. Other approaches will likely require more data and more assumptions. There is, however, sound guidance in the cases that while damages may not be based on speculation, "they need not be proved with

⁵² *Grain Processing Corp. v. Am. Maize-Prods. Co.*, 893 F. Supp. 1386, 1389-90, 37 U.S.P.Q.2d (BNA) 1299, 1302 (N.D. Ind. 1995).

⁵³ For an analysis of the but-for market that does not rely on proportionality, see Epstein, *supra* note 14.

⁵⁴ The author has recently developed a model that can be used for this purpose. See Roy J. Epstein & Daniel L. Rubinfeld, *Merger Simulation: A Simplified Approach with New Applications*, 69 ANTITRUST L.J. 895 (2002) (discussing deviations from "market share" proportionality).

unerring precision, either.”⁵⁵ Even if more complicated damages methodologies are put forward in a particular case, PERLS will still serve as a valuable reference point for evaluating the alternative analyses.

C. *The Split Award*

Finally, the economics of PERLS indicate that a split award, i.e., a reasonable royalty in addition to lost profits on infringing sales in the same market, probably is not appropriate in conjunction with price erosion. Lost profits calculated with PERLS are already supposed to make the patent holder whole, and an additional royalty is likely to result in overcompensation. The split award possibly has a better economic rationale when there is no price erosion claim. Economics predicts that price erosion should be endemic in infringements, and the reasonable royalty component can be viewed as a rough substitute for “missing” price erosion.⁵⁶ PERLS accounts for price erosion in a readily quantifiable manner, and this analysis should be preferable to the “legal fiction” that often characterizes reasonable royalty rates.⁵⁷ Moreover, PERLS is much easier to implement because it

⁵⁵ *Minnesota Mining & Mfg. Co. v. Johnson & Johnson Orthopaedics, Inc.*, 976 F.2d 1559, 1579, 24 U.S.P.Q.2d (BNA) 1321, 1338 (Fed. Cir. 1992).

⁵⁶ *In re Mahurkar Double Lumen Hemodialysis Catheter Patent Litig.*, 831 F. Supp. 1354, 1387, 28 U.S.P.Q.2d (BNA) 1801, 1827 (N.D. Ill. 1993) (“Patents create a lawful ability to exclude competition, and thus some ability to increase price.”), *aff’d*, 71 F.3d 1573, 37 U.S.P.Q.2d (BNA) 1138 (Fed. Cir. 1995). *See also* Jonathan I. Arnold et al., *The Law and Economics of Reasonable Royalty Damages After Black & Decker’s ‘Snakelight’ Litigation*, 7 FED. CIR. B.J. 373 (1997).

⁵⁷ *Panduit Corp. v. Stahl Bros. Fibre Works, Inc.*, 575 F.2d 1152, 1159, 197 U.S.P.Q. (BNA) 726, 732 (6th Cir. 1978). (“Determination of a ‘reasonable royalty’ after infringement, like many devices in the law, rests on a legal fiction . . . the ‘reasonable royalty’ device conjures a ‘willing’ licensor and licensee, who like Ghosts of Christmas Past, are dimly seen as ‘negotiating’ a ‘license.’”)

avoids the additional inquiry to determine an appropriate value for the reasonable royalty.

A reasonable royalty can be combined with lost profits when data are only available to determine lost profits in some but not all years of an infringement. The reasonable royalty can also come into play when there is a claim that the infringer “expanded” the market for the patent holder and therefore did not cause lost profits damages. One scenario that is sometimes used to justify this claim is that the infringer had a better marketing campaign and made the patented feature a “hit” with consumers, increasing the patent holder’s own sales. In this case, the patent holder’s sales could be lower in the but-for market, thereby lowering lost profits damages. This could be a situation, however, where the reasonable royalty exceeds lost profits, so that the royalty would become the appropriate compensation to the patent holder. Another market expansion scenario is that price erosion led to increased quantities sold due to the lower actual price working through the price elasticity. PERLS indicates, however, that this effect generally increases the amount of lost profits, not the other way around.

III. ESTABLISHING THE AMOUNT OF PRICE EROSION

The preceding analysis assumes the amount of price erosion is known. Many methods have been employed over the years to calculate price erosion. This section briefly summarizes the approaches used in one foundational case and several influential recent decisions. Then, a different method is presented based on economic theory, using the same assumptions that underlie the PERLS lost profits Equation (1).

The 1886 decision by the Supreme Court in *Yale Lock v. Sargent*⁵⁸ provides a remarkably insightful discussion of economic issues relevant to determining price erosion. It is also noteworthy as an early example of the

⁵⁸ 117 U.S. 536 (1886).

gulf that generally separates the damages calculations of the parties. The defendant asserted zero damages but the Supreme Court ultimately affirmed an award of \$7,771 for price erosion damages. The defendant pointed out many potentially relevant factors that could have influenced the plaintiff's prices independent of the infringement, including competition and entry by other firms, decreases in costs, and general reduction in demand:

The defendant contends that the competition of the defendant was not the sole cause of the reduction of the plaintiff's prices, and that the proportionate effect of the defendant's competition is not attempted to be estimated or ascertained by the proofs. It alleges that the defendant is not responsible for the reduction made in 1873; that there were many other causes which contributed to this reduction; and that the lowering of prices was caused principally by the competition of other fireproof safe lock-makers, and, notably, the New Britain Lock Co., by the fact that safe-makers were making and threatening to make their own safe locks, and by the general lowering of the prices of material and labor and the depression of business.⁵⁹

The defendant's observations are well taken but do not negate the possibility of price erosion. Regardless of the decline in prices due to other factors, the infringement tends to cause prices to be even *lower*.

A. *Case Precedents*

The patent holder's pre-infringement price trends often provide a plausible benchmark for price erosion. In *Brooktree v. Advanced Micro Devices* the court calculated price erosion based on the selling price of the same

⁵⁹ *Id.* at 551.

product before the infringer entered the market.⁶⁰ The plaintiff proved to the court's satisfaction, through its documents and marketing witnesses, that it had planned to lower prices by 10% per year over the claimed damages period, while its actual price reductions were on the order of 30% per year.⁶¹

There are pitfalls in comparing prices before and after the infringement. For example, it is important to take account of other potentially relevant market factors, such as actual or anticipated entry of non-infringing competitors, or reduced costs for the patent holder, which could lead to unilateral price reductions. In addition, price calculations in both periods should include the effect of free promotional goods and other discounts, such as free or extended warranties. Finally, changes in product specifications or quality would require additional adjustments before comparing prices.

Alternatively, evidence of price erosion of a more direct nature is sometimes available from price quotes and other sales records, especially when there is head-to-head competition with the defendant. There was testimony in *Brooktree* that the plaintiff was forced to lower its prices to prevent losing business it had already developed.⁶² In *Lam v. Johns-Manville Corp* there was evidence that competition from defendants on four specific jobs forced Lam to reduce its price from its customary level to meet defendants' price-cutting.⁶³

⁶⁰ *Brooktree Corp. v. Advanced Micro Devices, Inc.*, 977 F.2d 1555, 1579, 24 U.S.P.Q.2d (BNA) 1401, 1418 (Fed. Cir. 1992).

⁶¹ *Id.*

⁶² *Id.*

⁶³ *Lam, Inc. v. Johns-Manville Corp.*, 219 U.S.P.Q. (BNA) 599, 600 (D. Colo. 1982).

In addition to trends in the patent holder's prices, it may be appropriate to use price trends in comparable markets as evidence of price erosion. In *Minnesota Mining & Manufacturing v. Johnson & Johnson Orthopaedics*, the plaintiff relied on the pre-infringement prices and also used sales in another market as a benchmark for but-for price increases.⁶⁴ Actual prices declined steadily in the damages period.⁶⁵ The plaintiff claimed that but-for prices would have increased 4% per year based on marketing documents indicating a plan to keep pace with inflation.⁶⁶ The plaintiff also used evidence that the defendant was able to raise prices 4% per year in a comparable market for a different product.⁶⁷ The special master in the case concluded that the plaintiff would have taken annual price increases of 2%, so the resulting price erosion would be the difference between 2% and the actual price decline in each year.⁶⁸ The use of prices from a different market requires separate analysis to establish that the benchmark provides a reasonable comparison.⁶⁹

Judge Easterbrook's opinion in *Mahurkar* used an approach based on savings made possible by the patented product.⁷⁰ The plaintiff suggested its average catheter prices but-for the infringement would have increased 10%

⁶⁴ *Minnesota Mining & Mfg. Co. v. Johnson & Johnson Orthopaedics, Inc.*, 976 F.2d 1559, 1579, 24 U.S.P.Q.2d (BNA) 1321, 1338 (Fed. Cir. 1992).

⁶⁵ *Id.*

⁶⁶ *Id.*

⁶⁷ *Id.*

⁶⁸ *Id.*

⁶⁹ *Id.*

⁷⁰ *In re Mahurkar Double Lumen Hemodialysis Catheter Patent Litig.*, 831 F. Supp. 1354, 1389, 28 U.S.P.Q.2d (BNA) 1801, 1829 (N.D. Ill. 1993), *aff'd*, 71 F.3d 1573, 37 U.S.P.Q.2d (BNA) 1138 (Fed. Cir. 1995).

per year, from \$32.81 in 1986 to \$52.84 in 1991, compared to the actual rate of less than 2% per year.⁷¹ There was testimony that older types of catheters were more expensive than the plaintiff's product, with prices ranging up to \$90, and that the plaintiff's design was functionally superior and cheaper to implant.⁷² These considerations led to a finding that a 10% price increase per year was reasonable and more likely too low than too high.⁷³ Moreover, the decision cited a series of opinions that resolve uncertainty in patent damages calculations against the infringer.⁷⁴

B. *An Economic Model for Price Erosion*

The economic theory of demand provides a different framework for determining the amount of price erosion when the market share rule applies to the but-for market. The advantage of this approach is that it can be easily implemented in a wide range of circumstances with a minimum of data. The price erosion calculation requires only the incremental profit margin for the patent holder, the infringer's share, the patent holder's share, and the market price elasticity, information that should be readily available.⁷⁵ The model-based analysis of price erosion is also appealing as a transparent benchmark. When it differs significantly from the results of other methods, it may be a signal of errors in more complicated, alternative analyses.

⁷¹ *Id.*

⁷² *Id.*

⁷³ *Id.*

⁷⁴ *Id.* at 1388, 28 U.S.P.Q.2d (BNA) at 1828 (citing *Story Parchment Co. v. Paterson Parchment Paper Co.*, 282 U.S. 555, 562-66 (1931); *Del Mar Avionics, Inc. v. Quinton Instrument Co.*, 836 F.2d 1320, 1327 (Fed. Cir. 1987); *Gyromat Corp. v. Champion Spark Plug Co.*, 735 F.2d 549, 554-55 (Fed. Cir. 1984)).

⁷⁵ Epstein, *supra* note 14, at 368-69.

The theory known as the Almost Ideal Demand System is the basis for the economic model of price erosion presented in this article.⁷⁶ It is widely used by economists in other contexts involving legal issues and, more generally, the analysis of consumer behavior.⁷⁷ Moreover, its economic properties are arguably superior to other models of market demand that have been proposed, especially when the goods in the relevant market are differentiated.⁷⁸ This theory can also be implemented using the market share rule, i.e., setting each firm's but-for market share equal to its share of the non-infringing revenue.⁷⁹ Indeed, the market share rule leads to a considerable simplification of the theory and results in a direct formula for the price erosion caused by the infringing sales.

A simple example of a market with three single-brand firms will help explain the logic of the demand model. Denote the patent holder as firm 1, a non-infringing alternative as firm 2, and the infringer as firm 3. The model specifies that the share of a given firm, as a percent of total market revenue,

⁷⁶ See generally Angus Deaton & John Muellbauer, *An Almost Ideal Demand System*, 70 AM. ECON. REV. 312 (1980).

⁷⁷ The approach has been used extensively in antitrust for merger analysis. See, e.g., Jerry A. Hausman & Gregory K. Leonard, *Economic Analysis of Differentiated Products Mergers Using Real World Data*, 5 GEO. MASON L. REV. 321, 327 (1997); Atanu Saha & Peter Simon, *Predicting the Price Effect of Mergers with Polynomial Logit Demand*, 7 INT'L J. ECON. BUS. 149 (2000), available at <http://proquest.umi.com> (last visited Sept. 20, 2002); Epstein & Rubinfeld, *supra* note 54.

⁷⁸ See Roger D. Blair & Thomas F. Cotter, *Rethinking Patent Damages*, 10 TEX. INTELL. PROP. L.J. 1, 52 (2002) (providing an overview of Cournot and residual demand models for undifferentiated commodities); see also ADDANKI, *supra* note 43, (discussing the constant-elasticity (log-linear) model); Epstein, *supra* note 14 (presenting a Cournot model with differentiated goods).

⁷⁹ See generally Epstein & Rubinfeld, *supra* note 54.

depends on the natural logarithms of the prices of all of the firms in the relevant market. The patent holder's share would be given by:

$$s_1 = a_1 + b_{1,1} \ln(p_1) + b_{1,2} \ln(p_2) + b_{1,3} \ln(p_3) .$$

Similar "share equations," with different coefficients, would characterize each of the other firms. The "own-coefficient" $b_{1,1}$ measures the effect of the patent holder's own price (p_1) on its share. This coefficient should have a negative sign since an increase in a firm's price should, with all other prices held constant, reduce its share. The other b s specify the effects of the prices of other firms on the patent holder's share. For example, $b_{1,2}$ specifies the effect of an increase in firm 2's price on share 1 and similarly for $b_{1,3}$. These "cross-effect" coefficients are expected to be positive since the patent holder's share should increase if prices rise for the competing firms.⁸⁰ The use of the logarithmic function means that changes in shares are related to *percentage* changes in prices. The demand system yields an economically consistent analysis of the effects of price changes throughout the market.⁸¹

⁸⁰ See generally Deaton & Muellbauer, *supra* note 76. The a_1 coefficient summarizes the effects of all non-price factors that influence share; these factors are assumed not to change in the but-for market. The formulation of the demand model in this article simplifies the original Deaton and Muellbauer specification by suppressing the aggregate expenditure term. Epstein & Rubinfeld, *supra* note 54, at 889 n.15 ("This 'homotheticity' assumption is reasonable to the extent that changes in industry expenditure have no significant effects on share.").

⁸¹ As an example, suppose the share equation for the patent holder was $s_1 = 0.4 - 0.6\ln(p_1) + 0.3\ln(p_2) + 0.1\ln(p_3)$. Suppose the initial prices were $p_1 = \$1.00$, $p_2 = \$0.90$, and $p_3 = \$1.20$. The equation yields a share for the patent holder of 38.7% at these prices. Suppose p_1 increases by 5% to \$1.05 with no change in the other prices. The equation would yield a new share equal to 35.7%. As expected, the decrease in share of three percentage points is equal to 5% times the associated own-coefficient of -0.6. The change in p_1 would also affect the shares of the other firms through the associated cross-coefficients in the other share equations.

Remarkably, the market share rule applied to the demand system makes it possible to calculate the but-for price increase for the patent holder without explicit values for any of the coefficients in the share equations. Ordinarily, the demand system has to be calibrated using complex econometric techniques.⁸² The market share rule essentially causes the algebraic terms involving the unknown coefficients to cancel throughout the model, leading to a simple expression for the price erosion. The only information needed to use the model in this case is the profit margin percentage for the patent holder, the infringer's share, the patent holder's share, and the market price elasticity. If the market price elasticity is equal to -1 , then the calculation simplifies further and requires only the profit margin and the infringer's share. The key result is an equation that predicts the amount of price erosion. When the market price elasticity is equal to -1 and the infringement does not affect incremental costs, it follows (after algebraic simplifications) that the price erosion predicted by the demand system under the market share rule is equal to:

$$\text{price erosion} = \text{patent holder's profit margin \%} \times \frac{\text{infringer's market share}}{1 - \text{infringer's market share}}$$

Using the notation introduced above for the lost profits Equation (1), this result can be expressed algebraically as⁸³

$$\delta = \mu \frac{s_I}{1 - s_I} \quad (\text{Equation 2})$$

For example, suppose the patent holder had an incremental profit margin of 40%, the infringer had a market share of 25%, and that elasticity equals -1 . Equation (2) yields predicted price erosion of 13.3% (40% times 25% divided by 75%). Even a relatively small amount of price erosion can

⁸² Werden, *supra* note 14, at 318.

⁸³ See *infra* app. § III.D.

imply significant damages because the implied increase in but-for margins is multiplied by all of the patent holder's but-for sales.

Equation (2) implies that price erosion rises with increases in the patent holder's profit margin percentage, which reflects the potential for price erosion. Price erosion also rises with increases in the infringer's share, i.e., the market success of the infringement. There may be factors in the market that can lead to falling prices in a given year. Nonetheless, as previously indicated, the analysis implies that absent the infringement prices would not have fallen as much or would have risen instead.

In practice, it is advisable to assess the amount of price erosion using a variety of methods. No single approach is the best in all situations. Trend analysis, comparable market analysis, cost savings, and evaluation of marketing documents should always be considered, if the data are available. The price erosion formula derived from the theory of demand complements the other approaches and serves as an independent, transparent, and economically consistent benchmark. The formula therefore offers significant additional potential for ensuring the reliability of price erosion analyses.

IV. APPLICATIONS OF PERLS TO SELECTED DECISIONS

This section uses PERLS to revisit three interesting patent infringement cases that include claims of price erosion.⁸⁴ As illustrations, the analyses are based only on information reported in the opinions,

⁸⁴ *Micro Motion, Inc. v. Exac Corp.*, 761 F. Supp. 1420, 19 U.S.P.Q.2d (BNA) 1001 (N.D. Cal. 1991); *Minnesota Mining & Mfg. Co. v. Johnson & Johnson Orthopaedics, Inc.*, 1991 U.S. Dist. LEXIS 11451 (D. Minn. Apr. 30, 1991), *aff'd*, 976 F.2d 1559, 24 U.S.P.Q.2d (BNA) 1321 (Fed. Cir. 1992); *Crystal Semiconductor Corp. v. Tritech Microelectronics Int'l, Inc.*, 246 F.3d 1336, 1359, 57 U.S.P.Q.2d (BNA) 1953, 1966 (Fed. Cir. 2001).

supplemented with reasonable price elasticity assumptions. It is possible to gain significant economic insights into the awards using only the most salient facts of the cases.

A. Micro Motion, Inc. v. Exac Corp.

In *Micro Motion, Inc. v. Exac Corp.*,⁸⁵ Exac infringed Micro Motion's patents for mass flowmeters, which measure the mass flow rate of liquids. Micro Motion sought compensation for lost profits due to lost sales, lost profits due to price erosion, reasonable royalty, prejudgment interest, and willfulness.⁸⁶ In addition, the lost sales claim included convoyed sales of accessories.⁸⁷ While this case raises a number of economic issues, the discussion here will focus on the determination of the amount of price erosion and the implied adjustment for lost profits. Price erosion constituted the bulk of the damages award, amounting to \$14.6 million.⁸⁸ The total award (before prejudgment interest) was \$20.8 million.⁸⁹

The court reviewed a large amount of complex evidence on price erosion to develop its opinion. It considered testimony of Micro Motion's managers on pricing; Exac's market share, volume discount policy, and strategy of dropping the effective price of its meters by adding additional features but not increasing prices; Micro Motion's market share, sales, profits, growth, and selling costs; the level of prices and competition for

⁸⁵ *Micro Motion*, 761 F. Supp. at 1422, 19 U.S.P.Q.2d (BNA) at 1003.

⁸⁶ *Id.*

⁸⁷ *Id.* at 1428, 19 U.S.P.Q.2d (BNA) at 1008.

⁸⁸ *Id.* at 1435, 19 U.S.P.Q.2d (BNA) at 1014.

⁸⁹ *Id.*

different product models; entry by new competitors;⁹⁰ and expert testimony on the price elasticity of demand for flowmeters, among other factors.⁹¹ The court concluded that the percentage of price erosion suffered was 4% for 1985 and 8% for 1986–1990.⁹² The arithmetic average of the price erosion for the six years of the damages period is 7.3%.

There is enough information in the opinion to use the economic model for price erosion presented in this article as an alternative measure of price erosion. Exac's average market share was approximately 10.4% over the damages period.⁹³ The court found that Micro Motion had an incremental profit margin of 65%.⁹⁴ Assuming a price elasticity of demand of -1 , the price erosion using this share and margin information with Equation (2) is 7.5%. The opinion also indicated that Micro Motion's economic expert estimated the price elasticity of demand for flowmeters during a portion of the damages period to be approximately -0.25 .⁹⁵ Using this elasticity and a deduced average market share of 86.7% for Micro Motion with the more general formulation in appendix V.D. yields average price erosion of 38.1%. As expected, a smaller magnitude for the elasticity implies a greater ability to raise price and therefore higher price erosion.

The economic model for price erosion presented in this article is valuable for requiring a minimum of information to help to assess

⁹⁰ *Id.* at 1433, 19 U.S.P.Q.2d (BNA) at 1012.

⁹¹ *Id.* at 1424-25, 1431-33, 19 U.S.P.Q.2d (BNA) at 1005, 1010-12.

⁹² *Id.* at 1434, 19 U.S.P.Q.2d (BNA) at 1013.

⁹³ *Id.* at 1425, 19 U.S.P.Q.2d (BNA) at 1005. The opinion reports shares for Exac of 3% in 1985, 10% in 1986, 13% in 1987, 16% in 1988, and 10% in 1989.

⁹⁴ *Id.* at 1429-30, 19 U.S.P.Q.2d (BNA) at 1009-10.

⁹⁵ *Id.* at 1431, 19 U.S.P.Q.2d (BNA) at 1010.

competing damages claims. Equation (2) yields a result that is virtually identical to the conclusion reached by the court. Alternative assumptions about price elasticity will imply different amounts of price erosion. The results in this example, however, strongly suggest that price erosion was unlikely to be zero and that the court took a conservative position, as the expert testimony on price elasticity suggested even higher price erosion than was awarded. That is, the model provides additional, reasonable evidence that price erosion was a feature of the infringement and that it was material.

There are other economic issues of interest in this decision from the point of view of PERLS. It appears that the court made separate calculations of lost sales and price erosion without incorporating an adjustment for price elasticity on the lost sales.⁹⁶ Failure to include this adjustment could overstate damages significantly, as the *Crystal* court warned.⁹⁷ Equation (1) indicates that with an elasticity of -1 the adjustment could reduce damages by approximately \$9 million dollars. Micro Motion also received a split award, which may not have been appropriate to be made whole.

B. Minnesota Mining & Mfg. Co. v. Johnson & Johnson Orthopaedics

*Minnesota Mining & Manufacturing v. Johnson & Johnson Orthopaedics*⁹⁸ involved damages for infringement in a market for orthopedic casting tapes. The plaintiff, 3M, had an average market share of approximately 54.5% over the damages period, and the defendant had an average market share of

⁹⁶ *Id.* at 1423-24, U.S.P.Q.2d (BNA) at 1004-05.

⁹⁷ *See supra* notes 11-12 and accompanying text.

⁹⁸ 1991 U.S. Dist. LEXIS 11451 (D. Minn. Apr. 30 1991), *aff'd*, 976 F.2d 1559, 24 U.S.P.Q.2d (BNA) 1321 (Fed. Cir. 1992).

approximately 24.3%.⁹⁹ The special master in the case determined lost sales damages to be \$23.3 million using the market share rule and awarded price erosion damages of \$28.9 million.¹⁰⁰ More specifically, the special master found that price erosion damages for 3M's actual sales plus the full amount of the incremental volume from the market share rule would be \$38.7 million.¹⁰¹ This is the naive market share rule discussed in Part II. However, 3M recognized that it would lose some sales due to but-for price increases, and the special master accepted the plaintiff's calculation that this "market contraction effect" would reduce its lost profits damages by \$9.8 million. Excluding a reasonable royalty that the court also awarded on certain units not included in 3M's lost profit claims, the bottom line was total damages of \$52.2 million.¹⁰²

The special master's opinion is impressive for its grasp of the economic issues and the level of detail of the calculations. It contains lengthy original analysis as well as critical assessments of the testimony of expert witnesses in the areas of economics, econometrics, and cost accounting. Both the complexity of the lost sales analysis and the absence of supporting information on the market contraction effect, however, invite use of PERLS as a check on the overall conclusion and to identify the most important assumptions in the analysis.

PERLS yields "base case" damages of \$47.8 million, assuming a benchmark market price elasticity of -1 and using a profit margin of 41.1% and the average infringing share of 24.3% indicated in the opinion. This calculation incorporates price erosion of 13.2% predicted by Equation (2). These damages are within about 8% of the actual award, a small difference

⁹⁹ *Id.* at *132

¹⁰⁰ *Id.* at *143, *148.

¹⁰¹ *Id.* at *150.

¹⁰² *Id.* at *152.

that could be explained by the elasticity of -0.45 that appears to underlie the analysis in the opinion. Furthermore, the predicted price erosion stands in remarkably close agreement with the price erosion of 13.3% deduced from the opinion. It is striking that the formula's simple use of the patent holder's profit margin and the infringer's share extracts almost the same economic information as the opinion's complex discussion of trends in 3M's prices and industry producer price indexes, as well as different degrees of competitiveness of the infringer and the other firms in the market.¹⁰³

This example also shows how PERLS is useful as an independent test of the claims made by the experts for the parties. For example, the defendant's expert used regression analysis of visits to orthopedic surgeons to argue that the market contraction effect would have been \$60 to \$70 million instead of \$9.8 million.¹⁰⁴ A contraction of this magnitude would be extreme. PERLS supports the opinion of the special master not to accept the defendant's analysis.

C. **Crystal v. TriTech**

In *Crystal v. TriTech*,¹⁰⁵ Crystal sued for infringement of its patents for CODECs, which are analog-to-digital audio chips used in personal computers. The damages phase of this case has a complicated history, but the Federal Circuit ultimately accepted the original jury awards of \$11.8 million for lost sales and \$10 million for a reasonable royalty.¹⁰⁶ The Federal Circuit overturned the jury's award of \$26.6 million for price erosion, affirming the district court's judgment that the plaintiff's expert did not provide "substantial evidence to support Crystal's claims for any price

¹⁰³ *Id.* at *136-40, *143-51.

¹⁰⁴ *Id.* at *149-50.

¹⁰⁵ 246 F.3d 1336, 57 U.S.P.Q.2d (BNA) 1953 (Fed. Cir. 2001).

¹⁰⁶ *Id.* at 1345, 57 U.S.P.Q.2d (BNA) at 1956-57.

erosion at all.”¹⁰⁷ This case raises numerous issues concerning relevant market definition, measurement of price erosion, the interaction between price erosion and lost sales, and the role of the reasonable royalty.

“Crystal’s witnesses divided the overall audio chip market into two segments: the ‘low quality’ segment for chips below specific industry audio standards; and the ‘high quality’ segment for chips within those standards.”¹⁰⁸ The plaintiff claimed lost sales damages of \$14.3 million using the market share rule to calculate an average but-for share for Crystal of 41.9%.¹⁰⁹ The plaintiff’s lost sales damages were based only on shares within the high quality segment. Defendant TriTech’s expert included low quality chips in the market definition and thereby arrived at lost sales damages of only \$7.4 million using a much lower average but-for share for Crystal of 21.8%.¹¹⁰ The jury’s verdict of \$11.8 million in lost volume damages equates to a but-for market share of about 34.7% for Crystal (11.8 divided by 14.3 times 41.9%). The verdict is consistent with a relevant market defined as all high quality chips plus a fraction (about 23%) of the low quality segment, suggesting the jury found that at least some nominally low quality chips were reasonable substitutes for the products sold by the parties.

Crystal claimed price erosion ranging from approximately 10% to 22% using prices and margins for CODECs used in Apple computers as a benchmark.¹¹¹ The opinion, however, implies that Crystal claimed price erosion only on its actual sales: “the total CODEC units sold by Crystal between 1994 and 1998 would yield price erosion damages of

¹⁰⁷ *Id.* at 1358, 57 U.S.P.Q.2d (BNA) at 1965-66.

¹⁰⁸ *Id.* at 1354, 57 U.S.P.Q.2d (BNA) at 1963.

¹⁰⁹ *Id.*

¹¹⁰ *Id.* at 1355, 57 U.S.P.Q.2d (BNA) at 1963.

¹¹¹ *Id.* at 1357, 57 U.S.P.Q.2d (BNA) at 1965.

\$34,700,000.”¹¹² If this is the case, it is not clear why Crystal would not also claim price erosion on the incremental sales from TriTech. The defendant argued that price erosion was zero. As already mentioned, the jury found price erosion damages of \$26.6 million, which suggests price erosion of approximately 7.7%. By comparison, the price erosion implied by Equation (2) appears to be on the order of 3.4% (using an assumed profit margin of 40.0% for Crystal and a market share of 7.9% for TriTech inferred from the opinion).

PERLS can be used to analyze the jury award and the lost profits claimed by the plaintiff and defendant. The alternatives considered are:

1. **Plaintiff:** 41.9% but-for share, 10% price erosion;
2. **Defendant:** 21.8% but-for share, 0% price erosion;
3. **Jury:** 34.7% but-for share, 7.7% price erosion;
4. **Formula PE:** 34.7% but-for share, price erosion given by demand model.

In addition, it is necessary to assume a value for the market price elasticity. The comparison uses a base case of -1 and alternatives of -0.5 and -2 as sensitivity tests. The results are summarized in Table 1.

¹¹² *Id.* at 1358, 57 U.S.P.Q.2d (BNA) at 1965.

Table 1

Alternative Damages Scenarios in *Crystal*
(\$ millions)

Scenario	But-For Share	Price Erosion	Lost Profits		
			Market Price Elasticity:		
			-0.5	-1	-2
Plaintiff	41.9%	10%	\$43.9	\$35.2	\$17.8
Defendant	21.8%	0%	\$7.4	\$7.4	\$7.4
Jury	34.7%	7.7%	\$34.4	\$28	\$15.1
Formula PE	34.7%	See note	\$24.9	\$19.3	\$12.9
Actual Decision		0%	\$11.8 lost profits \$10 reasonable royalty		

Note: The given elasticities imply corresponding price erosion from the demand model equal to 4.4%, 3.4%, and 1.5%, respectively.

This analysis yields a consistent total jury award of \$27.9 million using a price elasticity of -1 and an award of \$34.4 million using a price elasticity of -0.5 . The jury's actual lost profits award of \$38.4 million (\$11.8 million plus \$26.6 million) could be justified if there was foundation for high price erosion and small elasticity. Overall, Table 1 demonstrates that lost profits are sensitive to the value of the price elasticity.

The remaining damages issue in this case involves the reasonable royalty of \$10 million. Interestingly, the \$21.8 million total actual damages award (before doubling for TriTech's willfulness) is close to the "Formula PE" case with price elasticity in the range of -0.5 to -1 . *Crystal* therefore also illustrates how the reasonable royalty tends to serve as a de facto substitute for a price erosion award.

V. CONCLUSION

The market share framework for lost volume damages established by the Federal Circuit in *State Industries* can be easily extended to include economically consistent price erosion awards. Economics principles teach that price erosion effects, though difficult to account for, are likely to be endemic in patent infringements with implications for both the amount of lost sales (due to price elasticity effects) and appropriate but-for profit margins. This article introduces PERLS as an integrated analysis of price erosion, lost sales, and changes in incremental costs that should yield more reliable lost profits calculations in the typical patent infringement litigation. The analysis also suggests that, when the goal is to make the patent holder whole, a correct price erosion award makes it unnecessary to award a reasonable royalty on any infringing sales in the same market.

In its recent decision in *Crystal*, the Federal Circuit made it clear that to obtain price erosion damages, a patentee must produce credible economic evidence to show the decrease in sales, if any, that would have occurred at higher but-for prices. PERLS is a suitable means to satisfy this requirement: It provides a specific, quantifiable adjustment to conventional market share damages that explicitly takes account of price elasticity.

The new approach provides an economic structure for fact-intensive analysis of particular cases. The key analytical components are: (1) market definition and (2) determination of the amount of price erosion, the market price elasticity, and incremental costs. Economics then provides relationships involving the elasticity, price erosion, shares, and profit margins that yield a solution. This analysis, long used in antitrust and merger cases, offers many insights into the issues raised in the patent damages area. For example, this article has explained how a standard economic theory of demand can be used to provide a value for the amount of price erosion in the lost profits calculation. Further refinement of PERLS allows analysis of certain fact situations in greater detail; however, even in its current state of development, PERLS already offers a principled and

reasonably transparent means to improve the measurement of complex lost profits damages claims in patent infringement litigation.

APPENDIX

This appendix presents the derivation of the PERLS lost profits formula and explains its relationship to the existing market share rule. It also shows the derivation of Equation (2) for the predicted amount of price erosion. Finally, the appendix offers proofs of various statements in the article concerning the properties of PERLS.

I. PRICE ELASTICITY AND CHANGES IN REVENUE

- A. Denote the market price elasticity of demand as ε . The percentage change in market revenue REV_M corresponding to a percentage change δ in the average market price is approximated by

$$(\Delta REV_M)/REV_M = (1+\varepsilon)\delta$$

where Δ denotes the change in the associated variable.¹¹³ In the special case of $\varepsilon = -1$, price erosion has no effect on market revenue.

- B. But-for market revenue REV_M^b equals the actual market revenue plus the change in revenue due to δ . It follows that

$$\begin{aligned} REV_M^b &= REV_M + REV_M(\Delta REV_M)/REV_M \\ &= REV_M (1 + (1+\varepsilon)\delta) \end{aligned}$$

- C. The patent holder's but-for revenue REV^b equals its but-for share multiplied by the but-for market revenue.

¹¹³ See, e.g., KATZ & ROSEN, *supra* note 21, at 74.

II. THE CONVENTIONAL MARKET SHARE RULE

- A. The patent holder has market share s and the infringer has market share s_I . The patent holder's revenue is REV , and the infringer's revenue is REV_I . The patent holder has incremental profit margin percent μ .
- B. Lost sales under the conventional market share rule are the patent holder's share of non-infringing sales multiplied by the infringer's revenue. Lost sales therefore equal $s/(1 - s_I)REV_I = s/(1 - s_I)s_I REV_M = s_I/(1 - s_I)REV$, showing that market share lost sales can be expressed equivalently in terms of the infringer's share and the patent holder's revenue.
- C. The patent holder's but-for market share under the conventional market share rule without price erosion is $s/(1 - s_I)$. This follows because its but-for revenue is $REV + s_I/(1 - s_I)REV = REV/(1 - s_I) = (s/(1 - s_I))REV_M$.
- D. Lost profits damages in the absence of price erosion equal $\mu s_I/(1 - s_I)REV$.
- E. The conventional market share rule with price erosion on actual and but-for sales implicitly assumes a price elasticity of demand equal to zero. Assume a two-firm market, so $s = (1 - s_I)$. By definition, total actual market revenue equals REV/s . The patent holder's but-for revenue would be $(1 + \delta)REV + (1 + \delta)s_I/(1 - s_I)REV = (1 + \delta)REV/s$. The patent holder accounts for the entire but-for market. Under appendix I.B, the implied price elasticity ε must equal 0.

III. THE GENERAL PERLS EQUATION FOR LOST PROFITS DAMAGES

- A. The patent holder's lost profits are defined in PERLS as the sum of the incremental profit on the lost (i.e., incremental)

sales plus lost profits on the actual sales. Price elasticity enters the analysis as a determinant of the amount of lost sales.

- B. For the patent holder, actual price is P , quantity sold is q , and incremental cost is c . The incremental profit margin percent μ is $(P-c)/P$. A superscript "b" denotes a but-for level for the associated variable. The patent holder's but-for incremental cost for incremental sales units, denoted c_1^b , differs from c by a factor γ_1 , so that $c_1^b = c(1 + \gamma_1)$. The patent holder's but-for incremental cost for the actual sales units, denoted c_2^b , differs from c by a factor γ_2 , so that $c_2^b = c(1 + \gamma_2)$.
- C. The patent holder's but-for market share equals $s/(1-s_1)$, consistent with the conventional market share rule in appendix II.C.
- D. The PERLS lost profits equation is

Lost Profits = But-For Profits – Actual Profits

$$\begin{aligned}
 &= (P^b - c_1^b)(q^b - q) + (P^b - P - (c_2^b - c))q \\
 &= ((P^b - c_1^b)/P^b)P^b(q^b - q) + ((P^b - P)/P - (c_2^b - c)/P)Pq \\
 &= (1 - (c/P)(1 + \gamma_1)/(1 + \delta))(REV^b - (1 + \delta)REV) + (\delta - \gamma_2 c/P)REV \\
 &= (1 - (1 - \mu)(1 + \gamma_1)/(1 + \delta))(s/(1 - s_1)REV_M(1 + (1 + \varepsilon)\delta) - (1 + \delta)REV) \\
 &\quad + (\delta - \gamma_2(1 - \mu))REV \\
 &= REV \left[(\mu - \gamma_1(1 - \mu)) \frac{s_1}{1 - s_1} + (\delta - \gamma_2(1 - \mu)) + \delta \frac{s_1}{1 - s_1} \right] \\
 &+ \varepsilon \frac{REV}{1 - s_1} \frac{\delta}{1 + \delta} (\delta + (\mu - \gamma_1(1 - \mu)))
 \end{aligned}$$

The second term in the last expression is the elasticity adjustment.

- E. With zero price erosion ($\delta=0$) and no cost effects ($\gamma_1, \gamma_2=0$) the PERLS lost profits formula becomes

$$\mu \frac{s_I}{1 - s_I} REV$$

which equals the conventional market share lost profits calculation without price erosion derived in appendix II.D. With price erosion but no cost effects, the formula simplifies to

$$\mu REV \frac{s_I}{1 - s_I} + \delta REV + \delta REV \frac{s_I}{1 - s_I} + \epsilon \frac{REV}{1 - s_I} \frac{\delta}{1 + \delta} (\delta + \mu).$$

IV. JUDGE EASTERBROOK'S METHOD IN *MAHURKAR* AS A SPECIAL CASE OF PERLS

The opinion assumes the price elasticity equals -1 and that the patent holder would have a 100% share in the but-for market. The actual price is P , the price erosion is δ , and the incremental cost is c . The patent holder has revenue REV , market share s , and incremental profit margin percent μ . Let s_I be the share of infringing sales. Since all of the competitors are infringers, $s = 1 - s_I$.

The elasticity implies that the actual market revenue REV_M equals the but-for market revenue. It follows that but-for market quantity equals REV_M divided by the but-for price $P(1+\delta)$. Lost profits can be written (after making the substitution $REV_M = REV/s$) as

$$[REV/s - cREV / (sP(1+\delta))] - \mu REV.$$

After algebraic manipulation, this expression can be rewritten in the form

$$\mu REV \frac{s_I}{1-s_I} + \delta REV + \delta REV \frac{s_I}{1-s_I} + \varepsilon \frac{\delta}{1+\delta} REV \frac{(\delta + \mu)}{1-s_I}$$

which is the PERLS expression for lost profits when $\varepsilon = -1$ and $\gamma_1, \gamma_2 = 0$. This establishes the equivalence of the two methods in this case.

V. PRICE EROSION AND THE ALMOST IDEAL DEMAND SYSTEM

A. In the Almost Ideal Demand System, the elasticity ε_p facing the patent holder equals $b_{pp}/s - 1 + s(\varepsilon + 1)$, where b_{pp} is the own-coefficient.¹¹⁴ The but-for elasticity ε_p^b equals $b_{pp}/s^b - 1 + s^b(\varepsilon + 1)$.

B. Using $s^b = s/(1-s_I)$, it follows that

$$\varepsilon_p^b = (1-s_I)(\varepsilon_p + 1 - s(1+\varepsilon)) - 1 + s/(1-s_I)(1+\varepsilon).$$

C. Assuming $\gamma = 0$, $\delta = (1 + 1/\varepsilon_p)/(1 + 1/\varepsilon_p^b) - 1$.¹¹⁵

D. By substitution it follows that $\delta = (\mu(1-z) - 1)/z$, where $z = (1-s_I)[1 - 1/\mu - s(1+\varepsilon)] + s(1+\varepsilon)/(1-s_I)$.

E. If $\varepsilon = -1$, the predicted price erosion reduces to $\delta = \mu s_I/(1-s_I)$.

¹¹⁴ Epstein & Rubinfeld, *supra* note 54, at 894. For simplicity, this analysis suppresses the aggregate expenditure terms in the full demand system.

¹¹⁵ *Id.* at 914.